Chapter 2. Econometric model for obtaining parameters of food supply and demand

1. Cereal and oil crop sector

(1) Input demand function of crop production

A farmer is assumed to produce four cereals, i.e., rice (*RI*), wheat (*WH*), maize (*MZ*), and other cereals (*XG*) in addition to two oil crops, i.e., soybeans (*SB*) and other oil crops (*XS*). The relevant inputs are land, labor, capital, and fertilizer. Labor and capital are fixed factors in this model.

Short-run profit is found by subtracting labor costs and capital user costs from the variable profit as

$$\pi^S = \pi^V - \sum_i (w_{L,i} X_{L,i} + w_{K,i} X_{K,i}), \tag{1-1}$$

where i is an index of crops including oil crops *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.

The variable profit maximization problem is

$$\text{max.} \;\; \pi^V = \sum_i p_i Q_i - \sum_i (w_{A,i} X_{A,i} + w_{V,i} X_{V,i}) \tag{1-2}$$

$$\text{s.t.} \;\; Q_i = f_{Q,i}(X_{A,i}, X_{V,i}, \overline{X_{L,i}}, \overline{X_{K,i}}) \qquad \forall i, \tag{1-3}$$

where π^S represents the short-run profit, π^V stands for the variable profit, p_i and Q_i respectively denote farm prices and production of the six crops, $X_{A,i}$ and $X_{V,i}$ respectively represent the planted area and fertilizer inputs of the six crops, and $\overline{X_{L,i}}$ and $\overline{X_{K,i}}$ respectively denote the labor and capital inputs of the six crops. These are fixed factors. In addition, $w_{A,i}$, $w_{V,i}$, $w_{L,i}$, and $w_{K,i}$ respectively represent land rents, fertilizer prices, wage rates, and capital user costs of the six modes of crop production.

Production functions (1-3) are specified as Cobb–Douglas type functions, as shown below.

$$Q_i = \alpha_{0i} X_{A,i}^{\alpha A,i} X_{V,i}^{\alpha V,i} \overline{X_{L,i}}^{\alpha L,i} \overline{X_{K,i}}^{\alpha K,i} \qquad \forall i \tag{1-4}$$

Solving the maximization problem with these constraints, the following Lagrangian function is set.

$$L = \sum_i p_i Q_i - \sum_i (w_{A,i} X_{A,i} + w_{V,i} X_{V,i}) + \sum_i \lambda_i \left(Q_i - \alpha_{0i} X_{A,i}^{\alpha A,i} X_{V,i}^{\alpha V,i} \overline{X_{L,i}}^{\alpha L,i} \overline{X_{K,i}}^{\alpha K,i} \right) \tag{1-5}$$

The first-order conditions of the function (1-5) for productions and inputs are given as shown below.

$$\frac{\partial L}{\partial Q_i} = p_i + \lambda_i = 0 \;\; \Rightarrow \lambda_i = -p_i \qquad \forall i \tag{1-6}$$

$$\frac{\partial L}{\partial X_{A,i}} = -w_{A,i} - \lambda_i \alpha_{0i} \alpha_{A,i} X_{A,i}^{\alpha A,i-1} X_{V,i}^{\alpha V,i} \overline{X_{L,i}}^{\alpha L,i} \overline{X_{K,i}}^{\alpha K,i} = 0 \;\; \forall i \tag{1-7}$$

$$\frac{\partial L}{\partial X_{V,i}} = -w_{V,i} - \lambda_i \alpha_{0i} \alpha_{V,i} X_{A,i}^{\alpha A,i} X_{V,i}^{\alpha V,i-1} \overline{X_{L,i}}^{\alpha L,i} \overline{X_{K,i}}^{\alpha K,i} = 0 \;\; \forall i \tag{1-8}$$

Substituting equations (1-6) into equations (1-7) and (1-8) gives the following equations.

$$p_i \alpha_{0i} \alpha_{A,i} X_{A,i}^{\alpha A,i-1} X_{V,i}^{\alpha V,i} \overline{X_{L,i}}^{\alpha L,i} \overline{X_{K,i}}^{\alpha K,i} = w_{A,i} \;\; \forall i \tag{1-9}$$

$$p_i \alpha_{0i} \alpha_{V,i} X_{A,i}^{\alpha A,i} X_{V,i}^{\alpha V,i-1} \overline{X_{L,i}}^{\alpha L,i} \overline{X_{K,i}}^{\alpha K,i} = w_{V,i} \;\; \forall i \tag{1-10}$$

Taking the natural logarithm of each side of equations (1-9) and (1-10) yields the following equations as

$$A_i + \ln \alpha_{A,i} + (\alpha_{A,i} - 1) \ln X_{A,i} + \alpha_{V,i} \ln X_{V,i} = \ln w_{A,i} \qquad \forall i \tag{1-11}$$

$$A_i + \ln \alpha_{V,i} + \alpha_{A,i} \ln X_{A,i} + (\alpha_{V,i} - 1) \ln X_{V,i} = \ln w_{V,i} \qquad \forall i \tag{1-12}$$

where

$$A_i = \ln p_i + \ln \alpha_{0i} + \alpha_{L,i} \ln \overline{X_{L,i}} + \alpha_{K,i} \ln \overline{X_{K,i}}, \forall i \tag{1-13}$$

After solving equations (1-11) and (1-12) for $\ln X_{A,i}$ and $\ln X_{V,i}$, one can multiply each side of equation (1-11) by $(\alpha_{V,i} - 1)$ as

$$(\alpha_{V,i} - 1) A_i + (\alpha_{V,i} - 1) \ln \alpha_{V,i} + (\alpha_{V,i} - 1)(\alpha_{A,i} - 1) \ln X_{A,i} + \alpha_{V,i} (\alpha_{V,i} - 1) \ln X_{V,i} = (\alpha_{V,i} - 1) \ln w_{A,i} \tag{1-14}$$

Then, multiplying each side of equation (1-12) by $\alpha_{V,i}$,

$$\alpha_{V,i} A + \alpha_{V,i} \ln \alpha_{V,i} + \alpha_{V,i} \alpha_{A,i} \ln X_{A,i} + \alpha_{V,i} (\alpha_{V,i} - 1) \ln X_{V,i} = \alpha_{V,i} \ln w_{V,i}. \tag{1-15}$$

Subtracting equation (1-15) from (1-14), $\ln X_{A,i}$ is obtained:

$$-A_i + (\alpha_{V,i} - 1) \ln \alpha_{A,i} - \alpha_{V,i} \ln \alpha_{V,i} + (\alpha_{V,i} - 1)(\alpha_{A,i} - 1) \ln X_{A,i} - \alpha_{V,i} \alpha_{A,i} \ln X_{A,i} = (\alpha_{V,i} - 1) \ln w_{A,i} - \alpha_{V,i} \ln w_{V,i}$$

$$-A_i + (\alpha_{V,i} - 1) \ln \alpha_{A,i} - \alpha_{V,i} \ln \alpha_{V,i} + (-\alpha_{V,i} - \alpha_{A,i} + 1) \ln X_{A,i} = (\alpha_{V,i} - 1) \ln w_{A,i} - \alpha_{V,i} \ln w_{V,i}$$

$$(1 - \alpha_{V,i} - \alpha_{A,i}) \ln X_{A,i} = A_i - (\alpha_{V,i} - 1) \ln \alpha_{A,i} + \alpha_{V,i} \ln \alpha_{V,i} + (\alpha_{V,i} - 1) \ln w_{A,i} - \alpha_{V,i} \ln w_{V,i}$$

$$\ln X_{A,i} = \frac{A_i + (1-\alpha_{V,i})\ln\alpha_{A,i} + \alpha_{V,i}\ln\alpha_{V,i}}{1-\alpha_{V,i}-\alpha_{A,i}}$$

$$\times\frac{-(1-\alpha_{V,i})\ln w_{A,i} - \alpha_{V,i}\ln w_{V,i}}{1-\alpha_{V,i}-\alpha_{A,i}} \quad (1\text{-}16)$$

Multiplying each side of equation (1-11) by $\alpha_{A,i}$, one obtains

$\alpha_{A,i}A_i + \alpha_{A,i}\ln\alpha_{A,i}, + \alpha_{A,i}(\alpha_{A,i},-1)\ln X_{A,i}$
$+\alpha_{A,i}\alpha_{V,i}\ln X_{V,i} = \alpha_{A,i}\ln w_{A,i}$. (1-17)

In addition, multiplying each side of equation (1-12) by $(\alpha_{A,i}-1)$ gives the following expression.

$(\alpha_{A,i}-1)A_i + (\alpha_{A,i}-1)\ln\alpha_{V,i}$
$+\alpha_{A,i}(\alpha_{A,i}-1)\ln X_{A,i} + (\alpha_{A,i}-1)(\alpha_{V,i}-1)\ln X_{V,i}$
$=(\alpha_{A,i}-1)\ln w_{V,i}$ (1-18)

By subtracting equation (1-17) from (1-18), $\ln X_{V,i}$ is obtained.

$-A_i - \alpha_{A,i}\ln\alpha_{A,i} + (\alpha_{A,i}-1)\ln\alpha_{V,i}$
$+(\alpha_{A,i}-1)(\alpha_{V,i}-1)\ln X_{V,i} - \alpha_{A,i}\alpha_{V,i}\ln X_{V,i}$
$=-\alpha_{A,i}\ln w_{A,i} + (\alpha_{A,i}-1)\ln w_{V,i}$
$-A_i - \alpha_{A,i}\ln\alpha_{A,i} + (\alpha_{A,i}-1)\ln\alpha_{V,i}$
$+(-\alpha_{A,i}-\alpha_{V,i}+1)\ln X_{V,i}$
$=-\alpha_{A,i}\ln w_{A,i} + (\alpha_{A,i}-1)\ln w_{V,i}$
$(1-\alpha_{A,i}-\alpha_{V,i})\ln X_{V,i}$
$=A_i + \alpha_{A,i}\ln\alpha_{A,i} - (\alpha_{A,i}-1)\ln\alpha_{V,i}$
$-\alpha_{A,i}\ln w_{A,i} + (\alpha_{A,i}-1),\ln w_{V,i}$

$$\ln X_{V,i} = \frac{A_i + \alpha_{A,i}\ln\alpha_{A,i} + (1-\alpha_{A,i})\ln\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}$$

$$\times\frac{-\alpha_{A,i}\ln w_{A,i} - (1-\alpha_{A,i})\ln w_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}} \quad (1\text{-}19)$$

Parameters of equations (1-16) and (1-19) can be replaced with the following parameters.

$$\ln\beta_{0A,i} = \frac{\ln\alpha_{0i} + (1-\alpha_{V,i})\ln\alpha_{A,i} + \alpha_{V,i}\ln\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}} \quad (1\text{-}20)$$

$$\ln\beta_{0V,i} = \frac{\ln\alpha_{0i} + \alpha_{A,i}\ln\alpha_{A,i} + (1-\alpha_{A,i})\ln\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}} \quad (1\text{-}21)$$

Then, the following input demand functions of land and fertilizer are obtained after substituting parameters (1-20) and (1-21) and equation (1-13) into equations (1-16) and (1-19).

$$\ln X_{A,i} = \ln\beta_{0A,i} + \frac{1}{1-\alpha_{A,i}-\alpha_{V,i}}\ln p_i$$

$$-\frac{1-\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{A,i} - \frac{\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{V,i}$$

$$+\frac{\alpha_{L,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{L,i}} + \frac{\alpha_{K,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{K,i}} \quad (1\text{-}22)$$

$$\ln X_{V,i} = \ln\beta_{0V,i} + \frac{1}{1-\alpha_{A,i}-\alpha_{V,i}}\ln p_i$$

$$-\frac{\alpha_{A,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{A,i} - \frac{1-\alpha_{A,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{V,i}$$

$$+\frac{\alpha_{L,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{L,i}} + \frac{\alpha_{K,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{K,i}} \quad (1\text{-}23)$$

The input demand functions of land and fertilizer for rice, wheat, maize, other cereals, soybeans, and other oil crops are functions in which suffixes of functions (1-22) and (1-23) are replaced with *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.

Substituting production functions (1-4) into equations (1-9) and (1-10), respectively, gives the following parameters of production functions.

$$\alpha_{A,i} = \frac{w_{A,i}X_{A,i}}{p_iQ_i}, \quad \alpha_{V,i} = \frac{w_{V,i}X_{V,i}}{p_iQ_i} \qquad \forall i \quad (1\text{-}24)$$

The parameters of input demand functions (1-22) and (1-23) are calculated from these parameters of production functions. However, the parameters of labor and capital, which are fixed factors, are not obtained through this procedure.

(2) Supply function of crops

The supply function is obtained by substituting input demand functions into the production function. The production function produced by taking the logarithm of the Cobb–Douglas short-run production function (1-4) is the following.

$$\ln Q_i = \ln\alpha_{0i} + \alpha_{A,i}\ln X_{A,i} + \alpha_{V,i}\ln X_{V,i}$$

$$+\alpha_{L,i}\ln\overline{X_{L,i}} + \alpha_{K,i}\ln\overline{X_{K,i}} \quad (1\text{-}25)$$

Substituting the input demand function of land (1-22) and that of fertilizer (1-23) into (1-25) yields the equation presented below.

$$\ln Q_i = \ln\alpha_{0i}$$

$$+\alpha_{A,i}\left(\ln\beta_{0A,i}+\frac{1}{1-\alpha_{A,i}-\alpha_{V,i}}\ln p_i\right.$$

$$-\frac{1-\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{A,i}-\frac{\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{V,i}$$

$$\left.+\frac{\alpha_{L,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{L,i}}+\frac{\alpha_{K,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{K,i}}\right)$$

$$+\alpha_{V,i}\left(\ln\beta_{0V,i}+\frac{1}{1-\alpha_{A,i}-\alpha_{V,i}}\ln p_i\right.$$

$$-\frac{\alpha_{A,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{A,i}-\frac{1-\alpha_{A,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{V,i}$$

$$\left.+\frac{\alpha_{L,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{L,i}}+\frac{\alpha_{K,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{K,i}}\right)$$

$$+\alpha_{L,i}\ln\overline{X_{L,i}}+\alpha_{K,i}\ln\overline{X_{K,i}}$$

$$=\ln\alpha_{0,i}+\alpha_{A,i}\ln\beta_{0A,i}+\frac{\alpha_{A,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln p_i$$

$$-\frac{\alpha_{A,i}(1-\alpha_{V,i})}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{A,i}-\frac{\alpha_{A,i}\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{V,i}$$

$$+\frac{\alpha_{A,i}\alpha_{L,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{L,i}}+\frac{\alpha_{A,i}\alpha_{K,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{K,i}}$$

$$+\alpha_{V,i}\ln\beta_{0V,i}+\frac{\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln p_i$$

$$-\frac{\alpha_{V,i}\alpha_{A,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{A,i}-\frac{\alpha_{V,i}(1-\alpha_{A,i})}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{V,i}$$

$$+\frac{\alpha_{V,i}\alpha_{L,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{L,i}}+\frac{\alpha_{V,i}\alpha_{K,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln\overline{X_{K,i}}$$

$$+\alpha_{L,i}\ln\overline{X_{L,i}}+\alpha_{K,i}\ln\overline{X_{K,i}}$$

$$=\ln\alpha_{0i}+\alpha_{A,i}\ln\beta_{0A,i}+\alpha_{V,i}\ln\beta_{0V,i}$$

$$+\frac{\alpha_{A,i}+\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln p_i$$

$$-\frac{\alpha_{A,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{A,i}-\frac{\alpha_{V,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\ln w_{V,i}$$

$$+\left[\alpha_{L,i}+\frac{\alpha_{A,i}\alpha_{L,i}+\alpha_{V,i}\alpha_{L,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\right]\ln\overline{X_{L,i}}$$

$$+\left[\alpha_{K,i}+\frac{\alpha_{A,i}\alpha_{K,i}+\alpha_{V,i}\alpha_{K,i}}{1-\alpha_{A,i}-\alpha_{V,i}}\right]\ln\overline{X_{K,i}}$$

$$=\ln\alpha_{0,i}+\alpha_{A,i}\ln\beta_{0A,i}+\alpha_{V,i}\ln\beta_{0V,i}$$

$$+\frac{\alpha_{A,i}+\alpha_{V,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}\ln p_i$$

$$-\frac{\alpha_{A,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}\ln w_{A,i}-\frac{\alpha_{V,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}\ln w_{V,i}$$

$$+\frac{\alpha_{L,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}\ln\overline{X_{L,i}}+\frac{\alpha_{K,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}\ln\overline{X_{K,i}} \tag{1-26}$$

Equation (1-26) is the supply function of crop *i*. The supply functions of rice, wheat, maize, other cereals, soybeans, and other oil crops are functions in which suffixes of function (1-26) are replaced with *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.

(3) Supply response to changes in prices of other crops

A farmer is assumed to have a plan for planting rice (*RI*), wheat (*WH*), maize (*MZ*), other cereals (*XG*), soybeans (*SB*), and other oil crops (*XS*), respectively, on land areas $X_{A,RI}$, $X_{A,WH}$, $X_{A,MZ}$, $X_{A,XG}$, $X_{A,SB}$, and $X_{A,XS}$. Next, it is assumed that the farmer will expand the planted area of crop *j* by an amount equal to $\Delta X_{A,j}$. In this case, the elasticities of land demand for the output price of crop *j* are approximated as

$$\frac{\partial\ln X_{A,j}}{\partial\ln p_i}=\frac{\partial X_{A,j}}{\partial p_i}\frac{p_i}{X_{A,j}}\approx\frac{\Delta X_{A,j}}{\Delta p_i}\frac{p_i}{X_{A,j}}\quad\forall i,j, \tag{1-27}$$

where *i* is an index of crops such as *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.

If the total planted area is constant, then the increase in planted area of crop *j*, i.e., $\Delta X_{A,j}$, is expected to be equal to the sum of the decrease in planted areas of other crops: $\Delta X_{A,i\neq j}$.

$$\sum_{i\neq j}\Delta X_{A,i}=-\Delta X_{A,j},\ \forall j \tag{1-28}$$

Multiplying each side of equation (1-28) by $p_j/\Delta p_j$ gives the equation presented below.

$$\sum_{i\neq j}\frac{\Delta X_{A,i}}{\Delta p_j}\frac{p_j}{X_{A,i}}X_{A,i}=-\frac{\Delta X_{A,j}}{\Delta p_j}\frac{p_j}{X_{A,j}}X_{A,j}\quad,\forall j \tag{1-29}$$

Substituting equation (1-27) into equation (1-29) gives the following equation of the relation of elasticities.

$$\sum_{i \neq j} \frac{\partial \ln X_{A,i}}{\partial \ln p_j} X_{A,i} = -\frac{\partial \ln X_{A,j}}{\partial \ln p_j} X_{A,j} \quad ,\forall j \tag{1-30}$$

Therein, j is an index of crops, i.e., *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.

The corresponding rate of decrease of the planted area of other crops is assumed to be same if the crop j output price is increased. The following equation presents this relation:

$$\frac{\dfrac{\Delta X_{A,i}}{X_{A,i}}}{\Delta p_j} = \frac{\dfrac{\Delta X_{A,k}}{X_{A,k}}}{\Delta p_j} \quad ,\forall i \neq j, k \neq j\ ,\ \forall j \tag{1-31}$$

After dividing each side of equation (1-31) by p_j, the equation of elasticities is

$$\frac{\partial \ln X_{A,i}}{\partial \ln p_j} = \frac{\partial \ln X_{A,k}}{\partial \ln p_j} \quad ,\forall i \neq j, k \neq j\ ,\ \forall j\ . \tag{1-32}$$

Substituting equations (1-32) into equations (1-30) produces the following equations of elasticities.

$$\frac{\partial \ln X_{A,i}}{\partial \ln p_j} = -\frac{\partial \ln X_{A,j}}{\partial \ln p_j} \frac{X_{A,j}}{\sum_{i \neq j} X_{A,i}} \ ,\forall i \neq j\ ,\ \forall j\ . \tag{1-33}$$

Using the above relation among elasticities, one can consider the planted area response of crops in the short run. The following two assumptions are used in this model.

(1) Total planted area is constant in the short run.

(2) Decreasing rates of planted area of the five crops corresponding to increased planted areas of another crop are equal.

By virtue of these assumptions, elasticities of supply for prices of other crops can be inferred. Elasticities of supply of rice for output prices of wheat, maize, other cereals, soybeans, and other oil crops are acquired from the production function (1-25) and equation (1-33) as

$$\frac{\partial \ln Q_i}{\partial \ln p_j} = \frac{\partial \ln Q_i}{\partial \ln X_{A,i}} \frac{\partial \ln X_{A,i}}{\partial \ln p_j}$$

$$= -\alpha_{A,i} \frac{\partial \ln X_{A,j}}{\partial \ln p_j} \frac{X_{A,j}}{X_{A,T} - X_{A,j}}\ ,\ \forall i \neq j, \forall j\ , \tag{1-34}$$

where $X_{A,T} = \sum_i X_{A,i}$.

Land rent, fertilizer price, labor input, and capital input for other crops are expected to affect the planted area of the crop. Nevertheless, this model does not incorporate these effects because the effects of cross-input prices and input quantities are slight.

(4) Planted area functions of crops

Production is obtained from the following identity.

$$Q_{i,t} = Y_{i,t} A_{i,t} \tag{1-35}$$

In that expression, Q_i represents production, Y_i denotes yield, A_i stands for the harvested area, i is the index of crops, and t denotes that the data are measured at time t.

In this model, the yield function addresses the technological change. The shock of climate change and the planted area function are equivalent to the supply function. This function is based on adaptive expectations, as developed by Nerlove (1956).

The planted area function, i.e., supply function, can be specified as the following linear function.

$$A_{it} = a_i + \sum_j b_{ij} p_{jt}^* + u_{it} \tag{1-36}$$

Therein, A_{it} signifies the planted area, which is equal to the harvested area in this model. In addition, j is a crop index for the six grains and oil crops, p_{jt}* represents the expected price of crop j, and u_{it} is an error component.

Adaptive expectation relies on the assumption that the update of the expectation responses to the previous error is

$$p_{jt}^* - p_{jt-1}^* = (1-\lambda)(p_{jt-1} - p_{jt-1}^*). \tag{1-37}$$

This equation can be rewritten as

$$p_{jt}^* - \lambda p_{jt-1}^* = (1-\lambda) p_{jt-1}. \tag{1-38}$$

Then, by multiplying λ to the one-year-lagged function of (1-36), one obtains the following function.

$$\lambda A_{it-1} = \lambda a_i + \sum_j \lambda b_{ij} p_{jt-1}^* + \lambda u_{it-1} \tag{1-39}$$

Subtracting function (1-39) from function (1-36) yields the following function.

$$A_{it} - \lambda A_{it-1} = a_i - a_i \lambda$$
$$+ \sum_j b_{ij} \left(p_{jt}^* - \lambda p_{jt-1}^* \right) + u_{it} - \lambda u_{it-1} \tag{1-40}$$

In addition, substituting equation (1-38) into equation (1-40) yields the equation presented below.

$$A_{it} = a(1-\lambda) + \lambda A_{it-1}$$
$$+ \sum_j b_{ij} (1-\lambda) p_{jt-1} + u_{it} - \lambda u_{it-1} \tag{1-41}$$

Therefore, the explanatory variables of the planted area function are the planted area of the prior year and prices of grains and oil crops prices of the prior year.

The planted area function in this model is specified based on linear function (1-41). Parameter $b_{ij}(1-\lambda)$ is calculated from the elasticities, which were presented in earlier sections. These are changed to slopes using the average numbers in the initial year:

$$\frac{\partial A_{it}}{\partial p_{jt-1}} = \frac{\partial \ln A_{it}}{\partial \ln p_{jt-1}} \frac{A_{itBase}}{p_{jtBase}}$$

The parameters of the prior year, i.e., λ , are assigned 0.8 as default values following those of the IFPSIM.

The planted area is assumed to be unaffected by climate variables because the forecasted values of rainfall in each country of the MIROC5 are not so varied by 2060.

(5) Summary of elasticities

Table 2-1-1 presents elasticities of supply for output prices of rice, wheat, maize, other cereals, soybeans, and other oil crops. Supply function (1-26), input demand functions (1-22) and (1-23), and cross price elasticities (1-34) are used to compile this table. Table 2-1-2 presents elasticities of supply for prices of variable inputs and for quantities of fixed inputs.

Tables 2-1-3 – 2-1-5 show elasticities of input demand. Elasticities of variable input demand in the short run are exhibited in Table 2-1-3. The variable inputs are land and fertilizer. Elasticities of conditional and normal fixed input demand in the long run are shown respectively in Tables 2-1-4 and 2-1-5. The fixed inputs are labor and capital.

Table 2-1-1. Elasticities of supply for output prices in the short run.

		Producer price	
		Rice (*RI*)	Wheat (*WH*)
Supply	RI	$\frac{\alpha_{A,RI}+\alpha_{V,RI}}{1-(\alpha_{A,RI}+\alpha_{V,RI})}$	$\frac{-\alpha_{A,RI}}{1-(\alpha_{A,WH}+\alpha_{V,WH})}\frac{X_{A,WH}}{X_{A,T}-X_{A,WH}}$
	WH	$\frac{-\alpha_{A,WH}}{1-(\alpha_{A,RI}+\alpha_{V,RI})}\frac{X_{A,RI}}{X_{A,T}-X_{A,RI}}$	$\frac{\alpha_{A,WH}+\alpha_{V,WH}}{1-(\alpha_{A,WH}+\alpha_{V,WH})}$
	MZ	$\frac{-\alpha_{A,MZ}}{1-(\alpha_{A,RI}+\alpha_{V,RI})}\frac{X_{A,RI}}{X_{A,T}-X_{A,RI}}$	$\frac{-\alpha_{A,MZ}}{1-(\alpha_{A,WH}+\alpha_{V,WH})}\frac{X_{A,WH}}{X_{A,T}-X_{A,WH}}$
	XG	$\frac{-\alpha_{A,XG}}{1-(\alpha_{A,RI}+\alpha_{V,RI})}\frac{X_{A,RI}}{X_{A,T}-X_{A,RI}}$	$\frac{-\alpha_{A,XG}}{1-(\alpha_{A,WH}+\alpha_{V,WH})}\frac{X_{A,WH}}{X_{A,T}-X_{A,WH}}$
	SB	$\frac{-\alpha_{A,SB}}{1-(\alpha_{A,RI}+\alpha_{V,RI})}\frac{X_{A,RI}}{X_{A,T}-X_{A,RI}}$	$\frac{-\alpha_{A,SB}}{1-(\alpha_{A,WH}+\alpha_{V,WH})}\frac{X_{A,WH}}{X_{A,T}-X_{A,WH}}$
	XS	$\frac{-\alpha_{A,XS}}{1-(\alpha_{A,RI}+\alpha_{V,RI})}\frac{X_{A,RI}}{X_{A,T}-X_{A,RI}}$	$\frac{-\alpha_{A,XS}}{1-(\alpha_{A,WH}+\alpha_{V,WH})}\frac{X_{A,WH}}{X_{A,T}-X_{A,WH}}$

Table 2-1-1. Elasticities of supply for output prices in the short run (continued).

		Producer price	
		Maize (*MZ*)	Other cereals (*XG*)
Supply	RI	$\frac{-\alpha_{A,RI}}{1-(\alpha_{A,MZ}+\alpha_{V,MZ})}\frac{X_{A,MZ}}{X_{A,T}-X_{A,MZ}}$	$\frac{-\alpha_{A,RI}}{1-(\alpha_{A,XG}+\alpha_{V,XG})}\frac{X_{A,XG}}{X_{A,T}-X_{A,XG}}$
	WH	$\frac{-\alpha_{A,WH}}{1-(\alpha_{A,MZ}+\alpha_{V,MZ})}\frac{X_{A,MZ}}{X_{A,T}-X_{A,MZ}}$	$\frac{-\alpha_{A,WH}}{1-(\alpha_{A,XG}+\alpha_{V,XG})}\frac{X_{A,XG}}{X_{A,T}-X_{A,XG}}$
	MZ	$\frac{\alpha_{A,MZ}+\alpha_{V,MZ}}{1-(\alpha_{A,MZ}+\alpha_{V,MZ})}$	$\frac{-\alpha_{A,MZ}}{1-(\alpha_{A,XG}+\alpha_{V,XG})}\frac{X_{A,XG}}{X_{A,T}-X_{A,XG}}$
	XG	$\frac{-\alpha_{A,XG}}{1-(\alpha_{A,MZ}+\alpha_{V,MZ})}\frac{X_{A,MZ}}{X_{A,T}-X_{A,MZ}}$	$\frac{\alpha_{A,XG}+\alpha_{V,XG}}{1-(\alpha_{A,XG}+\alpha_{V,XG})}$
	SB	$\frac{-\alpha_{A,SB}}{1-(\alpha_{A,MZ}+\alpha_{V,MZ})}\frac{X_{A,MZ}}{X_{A,T}-X_{A,MZ}}$	$\frac{-\alpha_{A,SB}}{1-(\alpha_{A,XG}+\alpha_{V,XG})}\frac{X_{A,XG}}{X_{A,T}-X_{A,XG}}$
	XS	$\frac{-\alpha_{A,XS}}{1-(\alpha_{A,MZ}+\alpha_{V,MZ})}\frac{X_{A,MZ}}{X_{A,T}-X_{A,MZ}}$	$\frac{-\alpha_{A,XS}}{1-(\alpha_{A,XG}+\alpha_{V,XG})}\frac{X_{A,XG}}{X_{A,T}-X_{A,XG}}$

Table 2-1-1. Elasticities of supply for output prices in the short run (continued).

		Producer price	
		Soybeans (*SB*)	Other oil crops (*XS*)
Supply	RI	$\frac{-\alpha_{A,RI}}{1-(\alpha_{A,SB}+\alpha_{V,SB})}\frac{X_{A,SB}}{X_{A,T}-X_{A,SB}}$	$\frac{-\alpha_{A,RI}}{1-(\alpha_{A,XS}+\alpha_{V,XS})}\frac{X_{A,XS}}{X_{A,T}-X_{A,XS}}$
	WH	$\frac{-\alpha_{A,WH}}{1-(\alpha_{A,SB}+\alpha_{V,SB})}\frac{X_{A,SB}}{X_{A,T}-X_{A,SB}}$	$\frac{-\alpha_{A,WH}}{1-(\alpha_{A,XS}+\alpha_{V,XS})}\frac{X_{A,XS}}{X_{A,T}-X_{A,XS}}$
	MZ	$\frac{-\alpha_{A,MZ}}{1-(\alpha_{A,SB}+\alpha_{V,SB})}\frac{X_{A,SB}}{X_{A,T}-X_{A,SB}}$	$\frac{-\alpha_{A,MZ}}{1-(\alpha_{A,XS}+\alpha_{V,XS})}\frac{X_{A,XS}}{X_{A,T}-X_{A,XS}}$
	XG	$\frac{-\alpha_{A,XG}}{1-(\alpha_{A,SB}+\alpha_{V,SB})}\frac{X_{A,SB}}{X_{A,T}-X_{A,SB}}$	$\frac{-\alpha_{A,XG}}{1-(\alpha_{A,XS}+\alpha_{V,XS})}\frac{X_{A,XS}}{X_{A,T}-X_{A,XS}}$
	SB	$\frac{\alpha_{A,SB}+\alpha_{V,SB}}{1-(\alpha_{A,SB}+\alpha_{V,SB})}$	$\frac{-\alpha_{A,SB}}{1-(\alpha_{A,XS}+\alpha_{V,XS})}\frac{X_{A,XS}}{X_{A,T}-X_{A,XS}}$
	XS	$\frac{-\alpha_{A,XS}}{1-(\alpha_{A,SB}+\alpha_{V,SB})}\frac{X_{A,SB}}{X_{A,T}-X_{A,SB}}$	$\frac{\alpha_{A,XS}+\alpha_{V,XS}}{1-(\alpha_{A,XS}+\alpha_{V,XS})}$

Note: $X_{A,T}=X_{A,RI}+X_{A,WH}+X_{A,MZ}+X_{A,XG}+X_{A,SB}+X_{A,XS}$

Table 2-1-2. Elasticities of supply for input prices and fixed inputs.

	Input price		Labor input	Capital input
	Land	Fertilizer		
Supply of crop *i*	$\frac{-\alpha_{A,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{-\alpha_{V,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{\alpha_{L,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{\alpha_{K,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$

Note: *i*: index of crops: *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.

Table 2-1-3. Elasticities of land and fertilizer input demand in the short run.

	Output price	Input price		Labor input	Capital input
		Land	Fertilizer		
Land input of crop *i*	$\frac{1}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{1-\alpha_{V,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{-\alpha_{V,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{\alpha_{L,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{\alpha_{K,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$
Fertilizer input of crop *i*	$\frac{1}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{-\alpha_{A,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{1-\alpha_{A,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{\alpha_{L,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$	$\frac{\alpha_{K,i}}{1-(\alpha_{A,i}+\alpha_{V,i})}$

Note: *i*: index of crops *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.

Table 2-1-4. Elasticity of conditional labor and capital input demand in the long run.

	Output	Input price			
		Land	Fertilizer	Labor	Capital
Labor input of crop *i*	$\frac{1}{\eta_i}$	$\frac{\alpha_{Ai}}{\eta_i}$	$\frac{\alpha_{Vi}}{\eta_i}$	$-\left(1-\frac{\alpha_{Li}}{\eta_i}\right)$	$\frac{\alpha_{Ki}}{\eta_i}$
Capital input of crop *i*	$\frac{1}{\eta_i}$	$\frac{\alpha_{Ai}}{\eta_i}$	$\frac{\alpha_{Vi}}{\eta_i}$	$\frac{\alpha_{Li}}{\eta_i}$	$-\left(1-\frac{\alpha_{Ki}}{\eta_i}\right)$

Note: *i*: index of crops *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.
$\eta_i=\alpha_{Ai}+\alpha_{Li}+\alpha_{Ki}+\alpha_{Vi}$

Table 2-1-5. Elasticity of labor and capital input demand in the long run.

	Output price	Input price			
		Land	Fertilizer	Labor	Capital
Labor input of crop i	$\frac{1}{1-\eta_i}$	$-\frac{\alpha_{Ai}}{1-\eta_i}$	$-\frac{\alpha_{Vi}}{1-\eta_i}$	$-\left(1+\frac{\alpha_{Li}}{1-\eta_i}\right)$	$-\frac{\alpha_{Ki}}{1-\eta_i}$
Capital input of crop i	$\frac{1}{1-\eta_i}$	$-\frac{\alpha_{Ai}}{1-\eta_i}$	$-\frac{\alpha_{Vi}}{1-\eta_i}$	$-\frac{\alpha_{Li}}{1-\eta_i}$	$-\left(1+\frac{\alpha_{Ki}}{1-\eta_i}\right)$

Note: i: index of crops *RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*.
$\eta_i = \alpha_{Ai} + \alpha_{Li} + \alpha_{Ki} + \alpha_{Vi}$

2. Meat, eggs, and milk products sector

(1) Feed input demand function of production of meats, eggs, and milk

A farmer is assumed to produce seven livestock products: beef (*BF*), sheep (*SH*), pork (*PK*), poultry (*PM*), other meats (*XM*), eggs (*EG*), and raw milk (*MK*). For that production, the inputs are rice for feed (*RI*, livestock), wheat for feed (*WH*, livestock), maize for feed (MZ, livestock), other grains for feed (*XG*, livestock), soybeans for feed (*SB*, livestock), other oil crops for feed (*XS*, livestock), soybean cake (*CS*, livestock), cake of other oil crops (*CX*, livestock), but also land (*A*), labor (*L*), and capital (*K*). In this model, land, labor, and capital are fixed factors.

The short-run profit is obtained by subtracting land rent, labor costs, and capital user costs from the variable profit as

$$\pi^S = \pi^V - \sum_i w_{A,i} X_{A,i} - \sum_i w_{L,i} X_{L,i} - \sum_i w_{K,i} X_{K,i} \qquad (2\text{-}1)$$

where i denotes *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, and *MK*, $w_{A,i}$ represents the land rent, $w_{L,i}$ stands for the wage rate, $w_{K,i}$ denotes capital user cost, $X_{A,i}$ denotes the land area, $X_{L,i}$ expresses labor input, $X_{K,i}$ signifies capital input, π^S is the short-run profit, and π^V is the variable profit.

The variable profit maximization problem is

$$\max.\ \pi^V = \sum_i p_i Q_i - \sum_i \sum_j p_j X_{j,i}\,, \qquad (2\text{-}2)$$

$$\text{s.t.}\quad Q_i = f_{Q,i}(X_{RI,i}, X_{WH,i}, X_{MZ,i}, X_{XG,i}, X_{SB,i}, X_{XS,i}, X_{CS,i}, X_{CX,i}, \overline{X_{A,i}}, \overline{X_{L,i}}, \overline{X_{K,i}}),\ \forall i \qquad (2\text{-}3)$$

where j can be *RI*, *WH*, *MZ*, *XG*, *SB*, *XS*, *CS*, or *CX*, and where p_i and Q_i denote farm prices and farm production quantities of the seven livestock products. In addtition, $X_{RI,i}$, $X_{WH,i}$, $X_{MZ,i}$, $X_{XG,i}$, $X_{SB,i}$, $X_{XS,i}$, $X_{CS,i}$, and $X_{CX,i}$ respectively stand for feed inputs of the seven livestock products. Also, $\overline{X_{A,i}}$, $\overline{X_{L,i}}$, and $\overline{X_{K,i}}$ respectively denote land, labor, and capital inputs of the seven livestock products. These are fixed factors.

The production functions (2-3) are specified as the Cobb–Douglas type function as

$$Q_i = \alpha_{0i} \prod_j X_{j,i}^{\alpha j,i}\, \overline{X_{A,i}}^{\alpha A,i}\, \overline{X_{L,i}}^{\alpha L,i}\, \overline{X_{K,i}}^{\alpha K,i}\quad \forall i, \qquad (2\text{-}4)$$

where i can represent *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, or *MK*. In addition, j can denote *RI*, *WH*, *MZ*, *XG*, *SB*, *XS*, *CS*, or *CX*.

Solving the maximization problem with these constraints, the following Lagrangian function is set.

$$L = \sum_i p_i Q_i - \sum_i \sum_j p_j X_{j,i} + \lambda_i \left(Q_i - \alpha_{0i} \prod_j X_{j,i}^{\alpha j,i}\, \overline{X_{A,i}}^{\alpha A,i}\, \overline{X_{L,i}}^{\alpha L,i}\, \overline{X_{K,i}}^{\alpha K,i} \right) \qquad (2\text{-}5)$$

In that equation, i can be *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, or *MK*. The first-order conditions of function (2-5) for production and inputs are

$$\frac{\partial L}{\partial Q_i} = p_i + \lambda_i = 0 \quad \Rightarrow \lambda_i = -p_i, \forall i, \qquad (2\text{-}6)$$

where i can be *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, or *MK*.

$$\frac{\partial L}{\partial X_{j,i}} = -p_j - \lambda_i \alpha_{0i} \alpha_{j,i} \prod_j X_{j,i}^{\alpha j,i} X_{j,i}^{-1}\, \overline{X_{A,i}}^{\alpha A,i}\, \overline{X_{L,i}}^{\alpha L,i}\, \overline{X_{K,i}}^{\alpha K,i} = 0, \forall j,i \qquad (2\text{-}7)$$

In that equation, i is *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, or *MK*; j can be *RI*, *WH*, *MZ*, *XG*, *SB*, *XS*, *CS*, or *CX*.

Substituting (2-6) into (2-7) produces the following equation.

$$p_i \alpha_{0i} \alpha_{j,i} \prod_j X_{j,i}^{\alpha j,i} X_{j,i}^{-1}\, \overline{X_{A,i}}^{\alpha A,i}\, \overline{X_{L,i}}^{\alpha L,i}\, \overline{X_{K,i}}^{\alpha K,i} = p_j \qquad (2\text{-}8)$$

Taking the logarithm of (2-8) and solving for each crop in the same manner as that used for the crop sector yields the input demand functions for feed. The feed input demand function of j^{th} crop for i^{th} livestock production is

$$\ln X_{j,i} = \ln \beta_{0j,i} + \frac{1}{1-\beta_i}\ln p_i - \left(1+\frac{\alpha_{j,i}}{1-\beta_i}\right)\ln p_j - \sum_{k\neq j}\frac{\alpha_{k,i}}{1-\beta_i}\ln p_k + \frac{\alpha_{A,i}}{1-\beta_i}\ln \overline{X_{A,i}} + \frac{\alpha_{L,i}}{1-\beta_i}\ln \overline{X_{L,i}} + \frac{\alpha_{K,i}}{1-\beta_i}\ln \overline{X_{K,i}} \tag{2-9}$$

where $\beta_i = \sum_j \alpha_{j,i}$.

The j^{th} crop and cake input demand function of the i^{th} livestock products is obtained by replacement from *j* to *RI*, *WH*, *MZ*, *XG*, *SB*, *XS*, *CS*, or *CX* and from *i* to *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, or *MK* in equation (2-9).

In the official statistics, crop supply for feed is not divided according to the type of livestock production. Therefore, the following aggregated feed input demand function is used in the model.

$$\sum_i \ln X_{j,i} = \sum_i \ln \beta_{0j,i} + \sum_i \frac{1}{1-\beta_i}\ln p_i - \sum_i\left[\left(1+\frac{\alpha_{j,i}}{1-\beta_i}\right)\ln p_j - \sum_{k\neq j}\frac{\alpha_{k,i}}{1-\beta_i}\ln p_k\right] + \sum_i\left(\frac{\alpha_{A,i}}{1-\beta_i}\ln \overline{X_{A,i}} + \frac{\alpha_{L,i}}{1-\beta_i}\ln \overline{X_{L,i}} + \frac{\alpha_{K,i}}{1-\beta_i}\ln \overline{X_{K,i}}\right) \tag{2-10}$$

(2) Supply function of meats, eggs, and milk

The supply function is obtained by substituting input demand functions into the production function. The production function for which the logarithm is taken for the Cobb–Douglas short-run production function of i^{th} livestock products is

$$\ln Q_i = \ln \alpha_{0i} + \sum_j \alpha_{j,i}\ln X_{j,i} + \alpha_{A,i}\ln \overline{X_{A,i}} + \alpha_{L,i}\ln \overline{X_{L,i}} + \alpha_{K,i}\ln \overline{X_{K,i}}, \tag{2-11}$$

where *i* can be *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, or *MK*, and where *j* can be *RI*, *WH*, *MZ*, *XG*, *SB*, *XS*, *CS*, or *CX*. In addition, $X_{j,i}$, $X_{A,i}$, $X_{L,i}$, and $X_{K,i}$ respectively denote feed, land, labor, and capital inputs for livestock product *i*.

Substituting the input demand functions of feed (2-9) into the production function (2-11) yields the equations presented below.

$$\begin{aligned}
\ln Q_i &= \ln \alpha_{0i} \\
&+\sum_j \alpha_{j,i}\ln \beta_{0j,i} + \frac{\sum_j \alpha_{j,i}}{1-\beta_i}\ln p_i \\
&-\sum_j \alpha_{j,i}\left[\left(1+\frac{\alpha_{j,i}}{1-\beta_i}\right)\ln p_j + \sum_{k\neq j}\frac{\alpha_{k,i}}{1-\beta_i}\ln p_k\right] \\
&+\sum_j \alpha_{j,i}\left(\frac{\alpha_{A,i}}{1-\beta_i}\ln \overline{X_{A,i}} + \frac{\alpha_{L,i}}{1-\beta_i}\ln \overline{X_{L,i}} + \frac{\alpha_{K,i}}{1-\beta_i}\ln \overline{X_{K,i}}\right) \\
&+\alpha_{A,i}\ln \overline{X_{A,i}} + \alpha_{L,i}\ln \overline{X_{L,i}} + \alpha_{K,i}\ln \overline{X_{K,i}} \\
&= \ln \alpha_{0i} + \sum_j \alpha_{j,i}\ln \beta_{0j,i} + \frac{\beta_i}{1-\beta_i}\ln p_i \\
&-\sum_j \alpha_{j,i}\left[\frac{1-\sum_{k\neq j}\alpha_{k,i}}{1-\beta_i}\ln p_j + \sum_{k\neq j}\frac{\alpha_{k,i}}{1-\beta_i}\ln p_k\right] \\
&+\frac{\beta_i\alpha_{A,i}}{1-\beta_i}\ln \overline{X_{A,i}} + \frac{\beta_i\alpha_{L,i}}{1-\beta_i}\ln \overline{X_{L,i}} + \frac{\beta_i\alpha_{K,i}}{1-\beta_i}\ln \overline{X_{K,i}} \\
&+\alpha_{A,i}\ln \overline{X_{A,i}} + \alpha_{L,i}\ln \overline{X_{L,i}} + \alpha_{K,i}\ln \overline{X_{K,i}} \\
&= \ln \alpha_{0i} + \sum_j \alpha_{j,i}\ln \beta_{0j,i} + \frac{\beta_i}{1-\beta_i}\ln p_i \\
&-\sum_j \frac{\alpha_{j,i}}{1-\beta_i}\ln p_j \\
&+\frac{\alpha_{A,i}}{1-\beta_i}\ln \overline{X_{A,i}} + \frac{\alpha_{L,i}}{1-\beta_i}\ln \overline{X_{L,i}} \\
&+\frac{\alpha_{K,i}}{1-\beta_i}\ln \overline{X_{K,i}}
\end{aligned} \tag{2-12}$$

In fact, equation (2-12) is the supply function of i^{th} livestock products. The explanatory variables of these supply functions of the livestock products are the output price, prices of crops and cakes, and the fixed factors: land, labor, and capital inputs.

(3) Head function of meats, eggs, and milk

The production is obtained from the following identity,

$Q_{it} = Y_{it} H_{it}$, (2-13)

where Q_i stands for production, Y_i denotes yield, i.e., production per head, H_i is the number of livestock, and i is an index of livestock products: *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, or *MK*.

In this model, the yield function incorporates the technological change. The head function is equivalent to the supply function. This function is based on the adaptive expectations similarly to the grain and oil crop section.

One can specify the planted area function, i.e., the supply function, to the following linear function as

$$H_{it} = a_i + b_i p_{it}^* + \Sigma_j c_{ij} p_{jt}^* + u_{it}, \quad (2\text{-}14)$$

where H_{it} represents the number of head of livestock, i is a livestock index for the seven domestic animals, p_{it}^* signifies the expected price of the animal i, j is a feed index, p_{jt}^* stands for the expected price of the feed j, and u_{it} is an error term.

Adaptive expectation relies on the assumption of the update of expectation responses to the previous error as

$$p_{jt}^* - p_{jt-1}^* = (1-\lambda)(p_{jt-1} - p_{jt-1}^*). \quad (2\text{-}15)$$

This equation can be rewritten as

$$p_{jt}^* - \lambda p_{jt-1}^* = (1-\lambda) p_{jt-1}. \quad (2\text{-}16)$$

Multiplying λ by the one-year-lagged function of (2-14) yields the following function.

$$\lambda H_{it-1} = \lambda a_i + b_i \lambda p_{it-1}^* + \sum_j c_{ij} \lambda p_{jt-1}^* + \lambda u_{it-1} \quad (2\text{-}17)$$

Then, subtracting function (2-17) from function (2-14) produces the following function.

$$H_{it} - \lambda H_{it-1} = a_i (1-\lambda) + b_i (p_{it}^* - \lambda p_{it-1}^*) + \sum_j c_{ij} \left(p_{jt}^* - \lambda p_{jt-1}^* \right) + u_{it} - \lambda u_{it-1} \quad (2\text{-}18)$$

Substituting equation (2-16) into equation (2-18) produces the following equation.

$$H_{it} = a_i (1-\lambda) + \lambda H_{it-1} + b_i (1-\lambda) p_{it-1} + \sum_j c_{ij} (1-\lambda) p_{jt-1} + u_{it} - \lambda u_{it-1} \quad (2\text{-}19)$$

Therefore, the explanatory variables of the head function are the number of head of the prior year, price of the animal of the prior year, grains, and prices of feed of the prior year.

(4) Summary of elasticities

Table 2-2-1 presents elasticities of feed input demand for output prices of the seven products, i.e., beef, sheep, pork, poultry, other meats, eggs, and raw milk and for input price of the six crops and two cakes, i.e., rice, wheat, maize, other cereals, soybeans, other oil crops, soybean cake and other cakes and for the three input quantities.

Table 2-2-2 shows elasticities of supply for input prices of the six crops and cakes. It includes the input quantities of land, labor, and capital.

Table 2-2-1. Elasticities of input demand for livestock production.

		Output price	Input price			
		Products i	Rice	Wheat	Maize	Other grain
Input demand	RI	$\frac{1}{1-\beta_i}$	$-1-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$
	WH	$\frac{1}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-1-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$
	MZ	$\frac{1}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-1-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$
	XG	$\frac{1}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-1-\frac{\alpha_{XG,i}}{1-\beta_i}$
	SB	$\frac{1}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$
	XS	$\frac{1}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$
	CS	$\frac{1}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$
	CX	$\frac{1}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$

Note: i, index of livestock products *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, and *XS*.

$\beta_i = \Sigma_j \alpha_{j,i}$, where j is *RI*, *WH*, *MZ*, *XG*, *SB*, *XS*, *CS*, and *CX*.

Table 2-2-1. Elasticities of input demand for livestock production (continued).

		Input price			
		Soybeans	Other oil crops	Soybean cake	Other cake
Input demand	RI	$-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$
	WH	$-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$
	MZ	$-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$
	XG	$-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$
	SB	$-1-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$
	XS	$-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-1-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$
	CS	$-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-1-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$
	CX	$-\frac{\alpha_{SB,i}}{1-\beta_i}$	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-1-\frac{\alpha_{CX,i}}{1-\beta_i}$

Table 2-2-1. Elasticities of input demand for livestock production (continued).

		Input quantity		
		Land	Labor	Capital
Input demand	Crop and cake *j*	$\frac{\alpha_{A,i}}{1-\beta_i}$	$\frac{\alpha_{L,i}}{1-\beta_i}$	$\frac{\alpha_{K,i}}{1-\beta_i}$

Table 2-2-2. Elasticities of supply for livestock products.

	Output price	Input price				
	Product *i*	Rice	Wheat	Maize	Other grain	Soybeans
Supply of livestock products *i*	$\frac{\beta_i}{1-\beta_i}$	$-\frac{\alpha_{RI,i}}{1-\beta_i}$	$-\frac{\alpha_{WH,i}}{1-\beta_i}$	$-\frac{\alpha_{MZ,i}}{1-\beta_i}$	$-\frac{\alpha_{XG,i}}{1-\beta_i}$	$-\frac{\alpha_{SB,i}}{1-\beta_i}$

Note: same as Table 2-2-1

Table 2-2-2. Elasticities of supply for livestock products (continued).

	Input price			Input quantity		
	Other oil crop	Soybean cake	Other cake	Land	Labor	Capital
Supply of livestock products *i*	$-\frac{\alpha_{XS,i}}{1-\beta_i}$	$-\frac{\alpha_{CS,i}}{1-\beta_i}$	$-\frac{\alpha_{CX,i}}{1-\beta_i}$	$\frac{\alpha_{A,i}}{1-\beta_i}$	$\frac{\alpha_{L,i}}{1-\beta_i}$	$\frac{\alpha_{K,i}}{1-\beta_i}$

3. Vegetable oil and cake sector

(1) Supply function of vegetable oils

A vegetable oil company is assumed to produce oil by investing soybeans or other oil crops, labor, and capital, where the latter two inputs are fixed factors. In this case, the short-run profit of soybean oil production is found by subtracting labor costs and capital user costs from the variable profit.

$$\pi_{OS}^{S} = \pi_{OS}^{V} - w_{L,OS}X_{L,OS} - w_{K,OS}X_{K,OS} \tag{3-1}$$

In that equation, $w_{L,OS}$ signifies the wage rate, $X_{L,OS}$ stands for labor input, $w_{K,OS}$ denotes capital user costs, and $X_{K,OS}$ represents the capital input.

Therefore, the variable profit maximization problem of soybean oil production is the following.

$$\text{max.} \quad \pi_{OS}^{V} = p_{OS}Q_{OS} - p_{SB}QD_{SB} \tag{3-2}$$

$$\text{s.t.} \quad Q_{OS} = f_{Q,OS}(QD_{SB}, \overline{X_{L,OS}}, \overline{X_{K,OS}}) \tag{3-3}$$

In those expressions, p_{OS} signifies the price of soybean oil, Q_{OS} expresses the soybean oil output, p_{SB} denotes the price of soybeans, QD_{SB} stands for supply of soybeans which is equivalent to the supply of processed products. Presumably, the production function is specified as Cobb–Douglas type.

$$Q_{OS} = \alpha_{0,OS}QD_{SB}^{\alpha SB,OS}\overline{X_{L,OS}}^{\alpha L,OS}\overline{X_{K,OS}}^{\alpha K,OS} \tag{3-4}$$

Solving the maximization problem with these constraints, the following Lagrangian function is set.

$$L = P_{OS}Q_{OS} - p_{SB}QD_{SB} + \lambda_{OS}\left(Q_{OS} - \alpha_{0,OS}QD_{SB}^{\alpha SB,OS} \times \overline{X_{L,OS}}^{\alpha L,OS}\overline{X_{K,OS}}^{\alpha K,OS}\right) \tag{3-5}$$

The first-order conditions are the following.

$$\frac{\partial L}{\partial Q_{OS}} = p_{OS} + \lambda_{OS} = 0 \ \Rightarrow \lambda_{OS} = -p_{OS} \tag{3-6}$$

$$\frac{\partial L}{\partial QD_{SB}} = -p_{SB} - \lambda_{OS}\alpha_{0,OS}\alpha_{SB,OS}QD_{SB}^{\alpha SB,OS-1} \times \overline{X_{L,OS}}^{\alpha L,OS}\overline{X_{K,OS}}^{\alpha K,OS} = 0 \tag{3-7}$$

Substituting (3-6) into (3-7), one obtains the following.

$$P_{OS}\alpha_{0,OS}\ \alpha_{SB,OS}\ QD_{SB}^{\alpha SB,OS-1} \times \overline{X_{L,OS}}^{\alpha L,OS}\overline{X_{K,OS}}^{\alpha K,OS} = p_{SB}$$

$$QD_{SB}^{1-\alpha SB,OS} = (\alpha_{0,OS}\alpha_{SB,OS})p_{OS}p_{SB}^{-1} \times \overline{X_{L,OS}}^{\alpha L,OS}\overline{X_{K,OS}}^{\alpha K,OS} \tag{3-8}$$

Taking the logarithm of (3-8), the soybean input demand function of soybean oil production is obtained as shown below.

$$(1-\alpha_{SB,OS})\ln QD_{SB} = \ln(\alpha_{0,OS}\alpha_{SB,OS}) + \ln p_{OS} - \ln p_{SB} + \alpha_{L,OS}\ln\overline{X_{L,OS}} + \alpha_{K,OS}\ln\overline{X_{K,OS}}$$

$$\ln QD_{SB} = \frac{\ln(\alpha_{0,OS}\alpha_{SB,OS})}{1-\alpha_{SB,OS}} + \frac{1}{1-\alpha_{SB,OS}}\ln p_{OS} - \frac{1}{1-\alpha_{SB,OS}}\ln p_{SB} + \frac{\alpha_{L,OS}}{1-\alpha_{SB,OS}}\ln\overline{X_{L,OS}} + \frac{\alpha_{K,OS}}{1-\alpha_{SB,OS}}\ln\overline{X_{K,OS}} \tag{3-9}$$

The other oil crops input demand function for other vegetable oil production is obtained similarly to soybean oil production. In this case, the production function is specified as presented below.

$$Q_{OX} = \alpha_{0,OX}QD_{XS}^{\alpha XS,OX}\overline{X_{L,OX}}^{\alpha L,OX}\overline{X_{K,OX}}^{\alpha K,OX} \tag{3-10}$$

In that expression, Q_{OX} represents the output of other vegetable oil, QD_{XS} represents the input of other oil crops, and $X_{L,OX}$ and $X_{K,OX}$ respectively denote the labor and capital inputs of other vegetable oil production. The other oil crop input demand function is the following.

$$\ln QD_{XS} = \frac{\ln(\alpha_{0,OX}\alpha_{XS,OX})}{1-\alpha_{XS,OX}} + \frac{1}{1-\alpha_{XS,OX}}\ln p_{OX} - \frac{1}{1-\alpha_{XS,OX}}\ln p_{XS} + \frac{\alpha_{L,OX}}{1-\alpha_{XS,OX}}\ln\overline{X_{L,OX}} + \frac{\alpha_{K,OX}}{1-\alpha_{XS,OX}}\ln\overline{X_{K,OX}} \tag{3-11}$$

As shown there, p_{OX} is price of other vegetable oils; p_{XS} is the price of other oil crops.

The supply functions of soybean oil and other vegetable oils are obtained by substituting input demand functions of soybeans and other oil crops into the oil production functions. First, taking the logarithm of production functions (3-4) and (3-10) yields the following functions.

$$\ln Q_{OS} = \ln \alpha_{0,OS} + \alpha_{SB,OS} \ln QD_{SB}$$

$$+\alpha_{L,OS} \ln \overline{X_{L,OS}} + \alpha_{K,OS} \ln \overline{X_{K,OS}} \qquad (3\text{-}12)$$

$$\ln Q_{OX} = \ln \alpha_{0,OX} + \alpha_{XS,OX} \ln QD_{XS}$$

$$+\alpha_{L,OX} \ln \overline{X_{L,OX}} + \alpha_{K,OX} \ln \overline{X_{K,OX}} \qquad (3\text{-}13)$$

Second, substituting input demand functions (3-9) and (3-11), respectively, into production functions (3-12) and (3-13) yields the following supply functions.

$$\ln Q_{OS} = \frac{\ln \alpha_{0,OS} + \alpha_{SB,OS} \ln \alpha_{SB,OS}}{1-\alpha_{SB,OS}}$$

$$+\frac{\alpha_{SB,OS}}{1-\alpha_{SB,OS}} \ln p_{OS} - \frac{\alpha_{SB,OS}}{1-\alpha_{SB,OS}} \ln p_{SB}$$

$$+\frac{\alpha_{L,OS}}{1-\alpha_{SB,OS}} \ln \overline{X_{L,OS}} + \frac{\alpha_{K,OS}}{1-\alpha_{SB,OS}} \ln \overline{X_{K,OS}}$$

(3-14)

$$\ln Q_{OX} = \frac{\ln \alpha_{0,OX} + \alpha_{XS,OX} \ln \alpha_{XS,OX}}{1-\alpha_{XS,OX}}$$

$$+\frac{\alpha_{XS,OX}}{1-\alpha_{XS,OX}} \ln p_{OX} - \frac{\alpha_{XS,OX}}{1-\alpha_{XS,OX}} \ln p_{XS}$$

$$+\frac{\alpha_{L,OX}}{1-\alpha_{XS,OX}} \ln \overline{X_{L,OX}} + \frac{\alpha_{K,OX}}{1-\alpha_{XS,OX}} \ln \overline{X_{K,OX}}$$

(3-15)

(2) Expectation model of supply of vegetable oil and oil cake

One can specify the supply function to the following linear function with expected prices as

$$Q_{Oit} = a_{Oi} + b_{Oi} p_{Oit}^* + c_{Oi} p_{Sit}^* + u_{Oi}, \qquad (3\text{-}16)$$

where Q_{Oit} denotes the supply of oil i, p_{Oit}^* represents the expected price of oil or oil cake i, p_{Sit}^* is the variable for the expected price of oil crop i, and u_{it} denotes error.

Adaptive expectation dictates that the expectation responses are updated by prior error as presented below.

$$p_{Oit}^* - p_{Oit-1}^* = (1-\lambda)(p_{Oit-1} - p_{Oit-1}^*) \qquad (3\text{-}17)$$

This equation can be rewritten as

$$p_{Oit}^* - \lambda p_{Oit-1}^* = (1-\lambda) p_{Oit-1}. \qquad (3\text{-}18)$$

The equation for oil crop i is derived similarly.

$$p_{Sit}^* - \lambda p_{Sit-1}^* = (1-\lambda) p_{Sit-1} \qquad (3\text{-}19)$$

By multiplying λ to the one-year-lagged function of (3-16), the following function is obtained.

$$\lambda Q_{Oit-1} = \lambda a_{Oi} + b_{Oi} \lambda p_{Oit-1}^* + \underline{c_{Oi}} \lambda p_{Sit-1}^* + \lambda u_{Oit-1}$$

(3-20)

Subtracting function (3-20) from function (3-16) yields the function presented below.

$$Q_{Oit} - \lambda Q_{Oit-1} = a_{Oi}(1-\lambda)$$
$$+ b_{Oi}(p_{Oit}^* - \lambda p_{Oit-1}^*)$$
$$+ c_{Oi}(p_{Sit}^* - \lambda p_{Sit-1}^*) + u_{Oit} - \lambda u_{Oit-1} \qquad (3\text{-}21)$$

Substituting equations (3-18) and (3-19) into equation (3-21) yields the equation presented below.

$$Q_{Oit} = a_{Oi}(1-\lambda) + \lambda Q_{Oit-1}$$
$$+ b_{Oi}(1-\lambda) p_{Oit-1}$$
$$+ c_{Oi}(1-\lambda) p_{Sit-1} + u_{Oit} - \lambda u_{Oit-1} \qquad (3\text{-}22)$$

Therefore, the explanatory variables of the supply function are the supply of the prior year, price of the vegetable oil of the prior year, and price of the oil crop of the prior year.

In the same manner, the supply function of oil cakes based on the adaptive expectation is

$$Q_{Cit} = a_{Ci}(1-\lambda) + \lambda Q_{Cit-1}$$
$$+ b_{Ci}(1-\lambda) p_{Cit-1}$$
$$+ c_{Ci}(1-\lambda) p_{Sit-1} + u_{Cit} - \lambda u_{Cit-1}, \qquad (3\text{-}23)$$

where Q_{Cit} stands for the supply of oil cake i, p_{Cit} denotes the price of oil cake i, and p_{Sit} represents the price of the oil crop.

(3) Summary of elasticities

Table 2-3-1 presents elasticities of the crop input demand of production of soybean and other vegetable oils for the output price, for the crop input price, and for the two inputs based on functions (3-9) and (3-11). Furthermore, Table 2-3-2 shows elasticities of vegetable oils supply for the output price, for the crop input price, and for the two inputs based on functions (3-14) and (3-15).

Table 2-3-1. Elasticity of input demand for vegetable oil.

	Oil output price	Crop input price	Labor input	Capital input
Crop input demand of soybean oil	$\frac{1}{1-\alpha_{SB,OS}}$	$-\frac{1}{1-\alpha_{SB,OS}}$	$\frac{\alpha_{L,OS}}{1-\alpha_{SB,OS}}$	$\frac{\alpha_{K,OS}}{1-\alpha_{SB,OS}}$
Crop input demand of other vegetable oils	$\frac{1}{1-\alpha_{XS,OX}}$	$-\frac{1}{1-\alpha_{XS,OX}}$	$\frac{\alpha_{L,OX}}{1-\alpha_{XS,OX}}$	$\frac{\alpha_{K,OX}}{1-\alpha_{XS,OX}}$

Table 2-3-2. Elasticity of supply for vegetable oil.

	Oil output price	Crop input price	Labor input	Capital input
Supply of soybean oil	$\frac{\alpha_{SB,OS}}{1-\alpha_{SB,OS}}$	$-\frac{\alpha_{SB,OS}}{1-\alpha_{SB,OS}}$	$\frac{\alpha_{L,OS}}{1-\alpha_{SB,OS}}$	$\frac{\alpha_{K,OS}}{1-\alpha_{SB,OS}}$
Supply of other vegetable oils	$\frac{\alpha_{XS,OX}}{1-\alpha_{XS,OX}}$	$-\frac{\alpha_{XS,OX}}{1-\alpha_{XS,OX}}$	$\frac{\alpha_{L,OX}}{1-\alpha_{XS,OX}}$	$\frac{\alpha_{K,OX}}{1-\alpha_{XS,OX}}$

4. Dairy products sector

Dairy product manufacturers are assumed to produce skimmed milk (*SK*), butter (butter and ghee) (*BT*), and cheese (*CH*) from raw milk (whole milk) (*MK*). Drinking milk is a food category of raw milk.

(1) Supply function of dairy products

A manufacturer of dairy products (*DP*: *SK*, *BT*, and *CH*) is assumed to produce butter by investing raw milk, labor, and capital. The latter two inputs are fixed factors. In this case, the short-run profit of dairy product production is found by subtracting labor costs and capital user costs from the variable profit.

$$\pi_i^S = \pi_i^V - w_{L,i}X_{L,i} - w_{K,i}X_{K,i} \tag{4-1}$$

In that equation, i is an index of dairy products: *SK*, *BT*, and *CH*. In addition, $w_{L,i}$ represents the wage rate, $w_{K,i}$ signifies the capital user cost, $X_{L,i}$ denotes the labor input, and $X_{K,i}$ stands for the capital input.

The variable profit maximization problem of dairy products production is

$$\text{max.} \quad \pi_i^V = p_iQ_i - p_{MK}QDP_{MK,i} \tag{4-2}$$

$$\text{s.t.} \quad Q_i = f_{Q,i}(QDP_{MK,i}, \overline{X_{L,i}}, \overline{X_{K,i}}), \tag{4-3}$$

where p_i denotes the dairy product price, Q_i stands for the dairy product output, p_i signifies the raw milk price, and $QDP_{MK,i}$ denotes the raw milk supply necessary for producing the dairy products. The production function is assumed to be Cobb–Douglas type.

$$Q_i = \alpha_{0,i} QDP_{MK,i}^{\alpha MK,i}\,\overline{X_{L,i}}^{\alpha L,i}\,\overline{X_{K,i}}^{\alpha K,i} \tag{4-4}$$

Solving the maximization problem with these constraints, the following Lagrangian function is set.

$$L = p_iQ_i - p_{MK}QDP_{MK,i} + \lambda_i\left(Q_i - \alpha_{0,i}QDP_{MK,i}^{\alpha MK,i}\,\overline{X_{L,i}}^{\alpha L,i}\,\overline{X_{K,i}}^{\alpha K,i}\right) \tag{4-5}$$

The first-order conditions are those presented below.

$$\frac{\partial L}{\partial Q_i} = p_i + \lambda_i = 0 \;\Rightarrow\; \lambda_i = -p_i \tag{4-6}$$

$$\frac{\partial L}{\partial QDP_{MK,i}} = -p_{MK} - \lambda_i\alpha_{0,i}\alpha_{MK,i}QDP_{MK,i}^{\alpha MK,i-1}\,\overline{X_{L,i}}^{\alpha L,i}\,\overline{X_{K,i}}^{\alpha K,i} = 0 \tag{4-7}$$

Substituting (4-6) into (4-7), one obtains the following.

$$p_i\alpha_{0,i}\alpha_{MK,i}QDP_{MK,i}^{\alpha MK,i-1}\,\overline{X_{L,i}}^{\alpha L,i}\,\overline{X_{K,i}}^{\alpha K,i} = p_{MK}$$

$$QDP_{MK,i}^{1-\alpha MK,i} = (\alpha_{0,i}\alpha_{MK,i})p_ip_{MK}^{-1}\,\overline{X_{L,i}}^{\alpha L,i}\,\overline{X_{K,i}}^{\alpha K,i} \tag{4-8}$$

Taking the logarithm of (4-8), the raw milk input demand function of the dairy products production is obtained as presented below.

$$(1-\alpha_{MK,i})\ln QDP_{MK,i} = \ln(\alpha_{0,i}\alpha_{MK,i}) + \ln p_i - \ln p_{MK} + \alpha_{L,i}\ln\overline{X_{L,i}} + \alpha_{K,i}\ln\overline{X_{K,i}}$$

$$\ln QDP_{MK,i} = \frac{\ln(\alpha_{0,i}\alpha_{MK,i})}{1-\alpha_{MK,i}} + \frac{1}{1-\alpha_{MK,i}}\ln p_i - \frac{1}{1-\alpha_{MK,i}}\ln p_{MK} + \frac{\alpha_{L,i}}{1-\alpha_{MK,i}}\ln\overline{X_{L,i}} + \frac{\alpha_{K,i}}{1-\alpha_{MK,i}}\ln\overline{X_{K,i}} \tag{4-9}$$

The supply functions of the dairy products are obtained by substituting the input demand function of raw milk into the production function of the dairy products. First, taking the logarithm of production functions (4-4), the related functions are obtained as presented below.

$$\ln Q_i = \ln\alpha_{0,i} + \alpha_{MK,i}\ln QDP_{MK,i} + \alpha_{L,i}\ln\overline{X_{L,i}} + \alpha_{K,i}\ln\overline{X_{K,i}} \tag{4-10}$$

Second, by substituting input demand functions (4-9) into the production functions (4-10), the supply function is obtained as presented below.

$$\alpha_{MK,i}\ln QDP_{MK,i}=\frac{\alpha_{MK,i}\ln(\alpha_{0,i}\alpha_{MK,i})}{1-\alpha_{MK,i}}$$
$$+\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\ln p_i-\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\ln p_{MK}$$
$$+\frac{\alpha_{MK,i}\alpha_{L,i}}{1-\alpha_{MK,i}}\ln\overline{X_{L,i}}+\frac{\alpha_{MK,i}\alpha_{K,i}}{1-\alpha_{MK,i}}\ln\overline{X_{K,i}}$$

$$\ln Q_i=\ln\alpha_{0,i}+\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}(\ln\alpha_{0,i}+\ln\alpha_{MK,i})$$
$$+\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\ln p_i-\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\ln p_{MK}$$
$$+\left(1+\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\right)\alpha_{L,i}\ln\overline{X_{L,i}}$$
$$+\left(1+\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\right)\alpha_{K,i}\ln\overline{X_{K,i}}$$

$$\ln Q_i=\frac{1}{1-\alpha_{MK,i}}\ln\alpha_{0,i}+\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\ln\alpha_{MK,i}$$
$$+\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\ln p_i-\frac{\alpha_{MK,i}}{1-\alpha_{MK,i}}\ln p_{MK}$$
$$+\frac{\alpha_{L,i}}{1-\alpha_{MK,i}}\ln\overline{X_{L,i}}+\frac{\alpha_{K,i}}{1-\alpha_{MK,i}}\ln\overline{X_{K,i}} \qquad (4\text{-}11)$$

(2) Expectations model of supply of dairy products

The supply function is specified as the following linear function with expected prices of

$$Q_{Dit}=a_{Di}+b_{Di}p_{Dit}^{*}+c_{Di}p_{MKt}^{*}+u_{Dit}, \qquad (4\text{-}12)$$

where Q_{Dit} represents the supply of dairy products Di, p_{Dit}^{*} denotes the expected price of dairy products Di, p_{MKt}^{*} signifies the expected price of raw milk, and u_{Dit} is the error term.

Adaptive expectations entail the assumption that the update of expectation responses to prior error occurs as

$$p_{Dit}^{*}-p_{Dit-1}^{*}=(1-\lambda)(p_{Dit-1}-p_{Dit-1}^{*}). \qquad (4\text{-}13)$$

This equation can be rewritten as

$$p_{Dit}^{*}-\lambda\, p_{Dit-1}^{*}=(1-\lambda)p_{Dit-1}. \qquad (4\text{-}14)$$

In the same manner, the following equation for raw milk is derived.

$$p_{MKt}^{*}-\lambda\, p_{MKt-1}^{*}=(1-\lambda)p_{MKt-1} \qquad (4\text{-}15)$$

Multiplying λ by the one year lagged function of (4-12) produces the following function.

$$\lambda\, Q_{Dit-1}=\lambda\, a_{Di}+b_{Di}\lambda\, p_{Dit-1}^{*}+\underline{c_{Di}}\,\lambda\, p_{MKt-1}^{*}+\lambda\, u_{Dit-1} \qquad (4\text{-}16)$$

Subsequently, subtracting function (4-16) from function (4-12) gives the following function.

$$Q_{Dit}-\lambda\, Q_{Dit-1}=a_{Di}(1-\lambda)+b_{Di}(p_{Dit}^{*}-\lambda\, p_{Dit-1}^{*})+c_{Di}(p_{MKt}^{*}-\lambda\, p_{MKt-1}^{*})+u_{Dit}-\lambda u_{Dit-1} \qquad (4\text{-}17)$$

Substituting equations (4-14) and (4-15) into equation (4-17) yields the equation presented below.

$$Q_{Dit}=a_{Di}(1-\lambda)+\lambda Q_{Dit-1}+b_{Di}(1-\lambda)p_{Dit-1}+c_{Di}(1-\lambda)p_{MKt-1}+u_{Dit}-\lambda u_{Dit-1} \qquad (4\text{-}18)$$

Therefore, the explanatory variables of the supply function are the supply of the prior year, the price of dairy products of the prior year, and the price of raw milk of the prior year.

(3) Summary of elasticities

Table 2-4-1 presents elasticities of input demand of milk production for the output price, for the milk price, and for the two inputs based on function (4-9). In the same manner, Table 2-4-2 shows elasticities of dairy product supply for output price, for milk price, and for the two inputs, based on function (4-11).

Table 2-4-1. Elasticity of input demand for raw milk.

	Dairy product output price	Raw milk input price	Labor input	Capital input
Raw milk input demand of dairy products	$\frac{1}{1-\alpha_{MK,DP}}$	$-\frac{1}{1-\alpha_{MK,DP}}$	$\frac{\alpha_{L,DP}}{1-\alpha_{MK,DP}}$	$\frac{\alpha_{K,DP}}{1-\alpha_{MK,DP}}$

DP: *SK*, *BT*, and *CH*

Table 2-4-2. Elasticity of supply for dairy products.

	Dairy product output price	Raw milk input price	Labor input	Capital input
Supply of dairy products	$\frac{\alpha_{MK,DP}}{1-\alpha_{MK,DP}}$	$-\frac{\alpha_{MK,DP}}{1-\alpha_{MK,DP}}$	$\frac{\alpha_{L,DP}}{1-\alpha_{MK,DP}}$	$\frac{\alpha_{K,DP}}{1-\alpha_{MK,DP}}$

DP: *SK*, *BT*, and *CH*

5. Demand sector

The consumption section is assumed to include 18 goods. Regarding the demand of agricultural products, retail prices of rice reflects consumption of steam rice after cooking purchased milled rice. However, no retail prices of wheat and other cereals exist in many cases because consumers eat those processed foods such as bread or noodles. Therefore, in consumer demand analyses, the substitutes of rice are not as wheat but bread or noodles. Retail prices of these processed foods are included as variables of the demand functions.

Results that are closer to the true figures will be obtained using the model if the processing stage of these crops is included in this model. However, if one were to include only the processing industry of wheat, then many related industries such as bread, noodles, flour, and cakes would also be included. The resultant model can be expected to become exceedingly complex. In consideration of this tendency, input demand functions of crops for the respective industries will be derived.

(1) Derivation of a short-run input demand function

Input demand functions of agricultural goods for food production can be derived. The demand function in this sector is the conditional input demand function, for which the food production is given because foods are necessity goods. They will be produced around a rational quantity. The variable cost minimization problem of a food producer is set as follows. Variable inputs of this model are six crops (*RI*, *WH*, *MZ*, *XG*, *SB*, and *XS*), two oils (*OS* and *OX*), five meats (*BF*, *SH*, *PK*, *PM*, and *XM*), eggs (*EG*), and four dairy products (*MK*, *SK*, *BT*, and *CH*). The fixed inputs are labor and capital.

$$C = C^V + w_L X_L + w_K X_K \tag{5-1}$$

$$\text{min.} \quad C^V = \sum_i p_i QDF_i \tag{5-2}$$

$$\text{s.t} \quad QF = f_{QF}(QDF_{RI}, \cdots, QDF_{CH}, X_L, X_K) \tag{5-3}$$

Therein, C signifies the total cost, C^V represents the variable cost, and i is an index of agricultural products for inputs of food production of *RI*, *WH*, *MZ*, *XG*, *SB*, *XS*, *OS*, *OX*, *BF*, *SH*, *PK*, *PM*, *XM*, *EG*, *MK*, *SK*, *BT*, and *CH*. In addition, p_i is an input price such as the wholesale price of the products, QDF_i is an input quantity of the products, QF represents dishes that are placed on a table, w_L and w_K respectively denote the wage rate and capital user costs, and X_L and X_K are labor and capital inputs for the food production company.

The production function (5-3) is specified as the following Cobb–Douglas type.

$$QF = \alpha_0 \prod_i QDF_i^{\alpha i} X_L^{\alpha XL} X_K^{\alpha XK} \tag{5-4}$$

Solving the minimization problem with these constraints, the following Lagrangian function is set.

$$L = \sum_i p_i QDF_i - \lambda\left(QF - \alpha_0 \prod_i QDF_i^{\alpha i} X_L^{\alpha XL} X_K^{\alpha XK} \right) \tag{5-5}$$

$$\frac{\partial L}{\partial QDF_j} = p_j - \lambda \alpha_0 \alpha_j QDF_j^{\alpha j - 1} \prod_{i \neq j} QDF_i^{\alpha i} X_L^{\alpha XL} X_K^{\alpha XK} = 0 \quad \forall j \tag{5-6}$$

Substituting (5-4) into (5-6), one obtains the following.

$$p_j = \lambda \alpha_j QDF_j^{-1} QF$$

$$\Rightarrow \lambda QF = \alpha_j^{-1} p_j QDF_j \qquad \forall j \tag{5-7}$$

Function (5-7) holds for $k \neq j$:

$$\alpha_k^{-1} p_k QDF_k = \alpha_j^{-1} p_j QDF_j$$

$$QDF_k = \alpha_j^{-1} \alpha_k p_j p_k^{-1} QDF_j \quad \forall k \neq j, \ \forall j \tag{5-8}$$

Substituting (5-8) into production function (5-4) produces the following function.

$$\begin{aligned} QF &= \alpha_0 \prod_i QDF_i^{\alpha i} X_L^{\alpha XL} X_K^{\alpha XK} \\ &= \left(\alpha_j^{-1} \alpha_{RI} p_j p_{RI}^{-1} QDF_j \right)^{\alpha RI} \\ &\quad \times \cdots \times \left(\alpha_j^{-1} \alpha_{CH} p_j p_{CH}^{-1} QDF_j \right)^{\alpha CH} \\ &\quad \times \alpha_0 X_L^{\alpha XL} X_K^{\alpha XK} \end{aligned}$$

$$\begin{aligned} QF &= QDF_j^{\alpha RI + \cdots + \alpha CH} p_j^{\alpha RI + \cdots + \alpha CH} \\ &\quad \times \alpha_j^{-\alpha RI - \cdots - \alpha CH} p_{RI}^{-\alpha RI} \cdots p_{CH}^{-\alpha CH} \\ &\quad \times \alpha_{RI}^{\alpha RI} \cdots \alpha_{CH}^{\alpha CH} \alpha_0 X_L^{\alpha XL}, X_K^{\alpha XK} \end{aligned}$$

Substituting $\beta = \alpha_{RI} + \cdots + \alpha_{CH}$ into the equation above, the following function is obtained.

$$QDF_j^{\beta} = QF \cdot p_j^{-\beta} \alpha_j^{\beta} p_{RI}^{\alpha RI} \cdots p_{CH}^{\alpha CH}$$

$$\times \alpha_{RI}^{-\alpha RI} \cdots \alpha_{CH}^{-\alpha CH} \alpha_0^{-1} X_L^{-\alpha XL} X_K^{-\alpha XK}$$

$$\begin{aligned}
QDF_j &= QF^{\frac{1}{\beta}} p_j^{-1} \alpha_j p_{RI}^{\frac{\alpha RI}{\beta}} \cdots p_{CH}^{\frac{\alpha CH}{\beta}} \\
&\quad \times \alpha_{RI}^{-\frac{\alpha RI}{\beta}} \cdots \alpha_{CH}^{-\frac{\alpha CH}{\beta}} \alpha_0^{-\frac{1}{\beta}} X_L^{-\frac{\alpha XL}{\beta}} X_K^{-\frac{\alpha XK}{\beta}} \\
&= QF^{\frac{1}{\beta}} p_{RI}^{\frac{\alpha RI}{\beta}} \cdots p_j^{\frac{\alpha j}{\beta}-1} \cdots p_{CH}^{\frac{\alpha CH}{\beta}} \\
&\quad \times \alpha_{RI}^{-\frac{\alpha RI}{\beta}} \cdots \alpha_j^{-\frac{\alpha j}{\beta}+1} \cdots \alpha_{CH}^{-\frac{\alpha CH}{\beta}} \\
&\quad \times \alpha_0^{-\frac{1}{\beta}} X_L^{-\frac{\alpha XL}{\beta}} X_K^{-\frac{\alpha XK}{\beta}} \\
&= QF^{\frac{1}{\beta}} p_{RI}^{\frac{\alpha RI}{\beta}} \cdots p_j^{-\frac{\beta-\alpha j}{\beta}} \cdots p_{CH}^{\frac{\alpha CH}{\beta}} \\
&\quad \times \alpha_{RI}^{-\frac{\alpha RI}{\beta}} \cdots \alpha_j^{\frac{\beta-\alpha j}{\beta}} \cdots \alpha_{CH}^{-\frac{\alpha CH}{\beta}} \\
&\quad \times \alpha_0^{-\frac{1}{\beta}} X_L^{-\frac{\alpha XL}{\beta}} X_K^{-\frac{\alpha XK}{\beta}} \quad \forall j
\end{aligned} \tag{5-9}$$

Table 2-5-1 presents elasticities of input demand for food of equation (5-9).

The parameters of the production functions are obtained by solving the profit maximization problem. The profit is

$$\pi = p_F \alpha_0 \prod_i QDF_i^{\alpha i} X_L^{\alpha XL} X_K^{\alpha XK} - \sum_i p_i QDF_i .$$

The first-order condition of the problem is

$$\frac{\partial \pi}{\partial QDF_i} = \alpha_i p_F \frac{QF}{QDF_i} - p_i = 0 .$$

Therefore,

$$\alpha_i = \frac{p_i QDF_i}{p_F QF} \qquad \forall i , \tag{5-10}$$

$$\beta = \sum_i \alpha_i = \frac{\sum_i p_i QDF_i}{p_F QF} , \tag{5-11}$$

where p_F represents the price of food.

Table 2-5-1 Conditional price elasticity of input demand.

		Output	Input price			
			RI	WH	⋯	CH
Input demand	RI	$\frac{1}{\beta}$	$-1+\frac{\alpha_{RI}}{\beta}$	$\frac{\alpha_{WH}}{\beta}$	⋯	$\frac{\alpha_{CH}}{\beta}$
	WH	$\frac{1}{\beta}$	$\frac{\alpha_{RI}}{\beta}$	$-1+\frac{\alpha_{WH}}{\beta}$	⋯	$\frac{\alpha_{CH}}{\beta}$
	⋮	⋮	⋮	⋮	⋱	⋮
	CH	$\frac{1}{\beta}$	$\frac{\alpha_{RI}}{\beta}$	$\frac{\alpha_{WH}}{\beta}$	⋯	$-1+\frac{\alpha_{CH}}{\beta}$